

Determining the Mass of GW170814 from Gravitational Wave Observations

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Abstract

In the year 1915, Einstein predicted the existence of gravitational waves as a consequence of his general theory of relativity. However, he feared they would be undetectable since they are very weak. A century later, gravitational waves were detected for the first time by the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo. The aim of this project was to study the properties of two merging black holes, detected by LIGO and Virgo on the 14 of August 2017. The first part of this study consisted of preprocessing and analysis of the LIGO signal data. The second part consisted of using the data in order to calculate the mass of the two merging black holes. The mean value of the masses was calculated to be $104 M_{\odot}$, within 385% of values determined in the existing literature.

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1 Introduction

In 2015, scientists from The Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo interferometer announced that gravitational waves had been detected for the first time [1]. Gravitational waves are ripples in spacetime that were predicted by Albert Einstein a century earlier as a consequence of his general theory of relativity [1]. The signals were named GW150914 and came from a collision of two black holes with a mass of approximately 29 and 36 solar masses respectively [1]. The strength of the gravitational wave decreases rapidly with distance, making them difficult to detect on Earth [2].

1.1 Gravitational Waves

In order to fully comprehend the paper a background of the topic will be given.

1.1.1 Special Theory of Relativity

From the end of the 17th century to the beginning of the 20th century, the prevailing model of the universe rested on Newton's three laws of motion: [3]

1. *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*
2. *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*
3. *To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

In order to resolve difficulties that arose from the Galilean transformations and the Newtonian principle of relativity, Einstein formulated a new principle of relativity. In the year 1905, Einstein published the special theory of relativity. This was inspired by Maxwell's equations, the Michelson-Morley experiment and the progress done within Riemannian geometry. The special theory of relativity was based on two postulates: [4]

1. *The laws of physics take the same form in all inertial frames of reference.* [5]
2. *Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body* [6].

A direct consequence is that the speed of light, c , is the same in all inertial frames of reference. This postulate results in seemingly paradoxical situations. One aspect of the special theory of relativity is that length becomes relative, i.e. it depends on the motion with respect to the frame of reference. If one were to imagine a stationary object moving in a frame of reference, S' , and an observer, O , in another frame of reference, S , the length of the object measured by an observer in S' would be l_0 . If the frame of reference, S , moves with a velocity v_x relative to S' the length of the object l will be measured as [4]

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}. \quad (1)$$

Since $v < c$, will $l < l_0$, the length of the moving object would be measured as shorter than the stationary one. The maximum length would be measured when the object is stationary relative to its frame of reference, which is called the proper length of the object. This phenomenon is called Lorentz-Fitzgerald contraction or length contraction. [4]

Another consequence of Einstein's special theory of relativity is that time is relative. It can be derived in a similar way as the Lorentz-Fitzgerald contraction and be expressed mathematically as [4]

$$t = t_0 \sqrt{1 - \frac{v^2}{c^2}}. \quad (2)$$

Thus $t < t_0$, time would be measured at the highest speed when it is stationary relative to its frame of reference, proper time. A clock in a moving frame of reference would tick slower for a person in a stationary frame of reference. This phenomenon is called time

dilation. [4]

1.1.2 The Minkowski Spacetime

Two years after the special theory of relativity was published, a mathematician named Hermann Minkowski proposed the Minkowski spacetime. Minkowski meant that Einstein's theory of special relativity would be best described in three dimensions of space and one dimension of time. Space and time would be combined in the non-Euclidean, four-dimensional spacetime. The general theory of relativity predicted the following about spacetime: [4]

1. The geodesic effect: Represents the effect of the curvature of spacetime.
2. The frame-dragging effect: A moving object will change its motion when moving close to another moving object.

Mass will curve spacetime, and this affects how other objects move. When a smaller object passes a larger object it will curve toward it. The curvature of spacetime results in what is normally called gravity[7]. Einstein's general theory of relativity resulted in predictions of gravitational waves (GW), ripples in spacetime that occur when an object accelerates in spacetime. However, they would be weak and difficult to detect, Einstein himself feared that they would be undetectable[8].

The ripples become measurable on earth during extremely energetic processes in space, for example merging black holes and binary pulsars. [4] Researchers at LIGO have divided gravitational waves into different categories, based on what is thought to generate them. These GW classes are continuous, stochastic, burst and compact binary inspiral. Continuous Gravitational Waves are produced by a single, spinning object, such as a neutron star. Since the rotation rate stays constant, the frequency and amplitude of the gravitational waves will stay constant. Stochastic Gravitational Waves are presumed to be the combined result of arbitrarily many smaller gravitational waves from undetermined sources. Burst gravitational waves are short-lived gravitational waves from unknown or

unanticipated sources. The Compact Binary Inspiral GWs are the type of GWs that LIGO and Virgo have detected so far. They are caused by orbiting pairs of supermassive objects, such as neutron stars and black holes. The sources of compact binary inspiral gravitational waves are divided into three subclasses:

1. Binary Neutron Star (BNS)
2. Binary Black Hole (BBH)
3. Neutron Star-Black Hole Binary (NSBH)

Each subclass generates a unique pattern of gravitational wave, despite the main principle being identical. [9]

When an object is affected by a gravitational wave it undergoes a tidal deformation and the object contracts which can be explained by the special theory of relativity. For instance, when Earth is affected by a gravitational wave, its length reduces. Gravitational waves change the proper distance between particles, this change in distance, strain, can be defined as the relationship between the variable l which stands for the proper distance and δl stands for the variation of l , [10]

$$h = \frac{\delta l}{l}, \quad (3)$$

The dimensionless variable h can be seen as a description of the amplitude of the GW. The value of h is typically around 10^{-21} , and to put it into perspective it could be compared to

$$10^{-21} \approx \frac{\text{Diameter of H atom}}{1 \text{ AU}}. \quad (4)$$

The variable h can also be described as the relationship between the mass of each black hole, M , and the distance to the collision in meters, D_{meter} , [11]

$$h \approx \frac{GM}{c^2} \cdot \frac{1}{D_{meter}} \cdot \left(\frac{v}{c}\right)^2. \quad (5)$$

The distance to GW170814 was approximately $1.67 \cdot 10^{25}$ meters, or 540_{-210}^{+130} megaparsecs. Megaparsecs are equivalent to $3.086 \cdot 10^{19}$ km but are often used as a unit for distances by astronomers. [12]

1.1.3 Black Holes

In a simplified model, a place in spacetime in which the gravitational potential

$$\frac{GM}{R} > c^2, \quad (6)$$

is called a black hole. Another description of a black hole is a body whose escape velocity is greater than c . Black holes are made when massive stars detonate as supernovas and form into stellar mass-black holes. If the black holes are close to one another, they can merge and form larger black holes. If two black holes merge it reaches a stage called binary black hole inspiral, a stage where they rotate around each other vigorously and create gravitational waves. Energy and angular momentum are carried away from the black holes, causing the orbit to shrink and the two black holes to merge. This is visualised in the figure 1. [13]

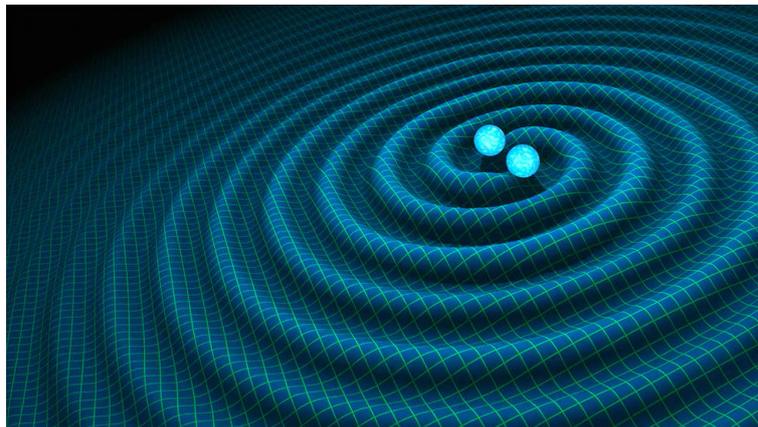


Figure 1: An artist's impression of a black hole inspiral generating ripples in spacetime [14].

1.1.4 Detecting Gravitational Waves

The first built laser interferometer¹ used to detect GWs is called the Laser Interferometer Gravitational-wave Observatory (LIGO). LIGO consists of two detectors, the first one is in Hanford, United States (H1), and the second one in Livingston, United States (L1). The reason for having two detectors is to confirm measurements. [15]

The principle is to send out two identical laser beams along the arms to a mirror that reflects the laser beams, and the changes in the length of the beam is measured. When the arm is not affected by a strong gravitational wave, the length of the light path remains unchanged and the light waves cancel each other out when they interchange. A gravitational wave causes spacetime to stretch, which means that one arm will extend and another one will contract when it moves from a peak to a trough. The signals are used to compare earlier values in order to detect gravitational waves. [15]

An example of a LIGO signal is shown in figure 2. In the raw data, it is in most cases not possible to see a clear signal visualising a peak indicating a black hole merger, furthermore, it is common that the signal is affected by other sources such as shaking of the ground, signals, and others. These other factors are called noise. In order to receive clear signals, such as the one presented in figure 2, the raw data has to be filtered. The filters used in this paper are called band-pass filter and a type of whitening, the power spectral density was also used as a part of the filtering.

¹The principle of an interferometer is that it works by merging sources of light to create an interference pattern.

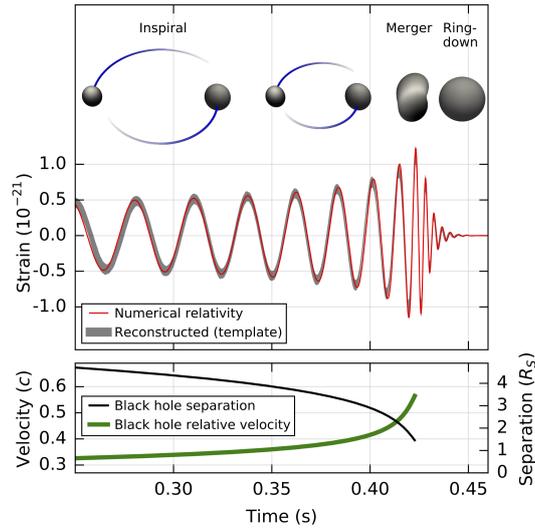


Figure 2: The x-axis represents the time and the y-axis represents the strain. The acceleration of merging black holes causes gravitational waves. As the black holes get closer to each other, they rotate faster and accelerate more, which increases the strain. When the two black holes merge, the acceleration stops and the strain and signal flattens out. [16]

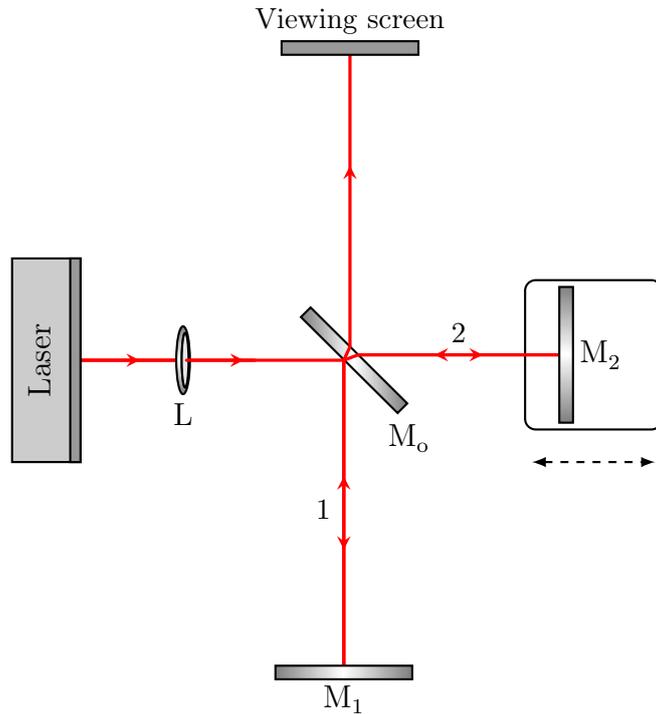


Figure 3: A laser beam is sent out from a laser source, split by a beam splitter and directed to two different mirrors, M_1 and M_2 . The two mirrors reflect the laser beams. A tidal deformation, gravitational wave, would result in the light waves being out of phase when interfering on the screen. [17]

1.2 Data Processing Methods

In order to clean the raw data multiple filters were used. The filters that were used are presented below.

1.2.1 Band-Pass filter

A band-pass filter (BPF) is a filter that can exclude frequencies in specific ranges, either eliminate frequencies lower than a cutoff frequency, a high pass filter (HPF), or higher, a low pass filter (LPF). [18]

1.2.2 Power Spectral Density

A composite signal can be divided into signals with single frequencies, and each frequency corresponds to a specific power level. The correlation between frequencies and power level can be specified by the power spectral density (PSD), generally a plot of the distribution of the average power of a frequency. A way of approximating the PSD is Welch's method. In Python, an estimate PSD is computed by splitting data into segments and computing a periodogram, estimation, of each segment. The average of the periodograms forms a PSD. [19]

1.2.3 Q-transform

A spectrogram is a way to visualise the frequency of a signal over time in a graph. It consists of a two-dimensional graph; the x-axis represents time, and the y-axis represents frequency. A third dimension representing a frequency or frequency bin at a particular time is visualised by a colour [20]. The constant-Q transform is similar to a spectrogram, it is made with short-time Fourier transforms (STFT), which has the advantage that the frequency bins are spaced more sparsely at higher frequencies and therefore easier to read [21].

1.3 Aim of Study

The aim of this study is firstly to process raw data of the gravitational waves caused by two merging black holes received by LIGO and Virgo on the 14th of August 2017 (GW170814)[1]. And secondly, to estimate the masses of the black holes.

2 Method

Raw signals from GW170814 were downloaded from `pycbc.catalog`, an open-source catalog established by PyCBC consisting of data from the Gravitational Wave Open Science Center (LIGO and Virgo). A Python code on GitHub, also established on PyCBC, was used to visualise the signals in plots [22]. The Python-code can be found in appendix A.

2.1 Data Algorithm

Frequencies of the raw data lower than 15 Hz were eliminated with a band-pass filter. By eliminating the low frequencies, frequencies caused by the detector could be removed.

An estimation of the PSD of the data was made with Welch's method. The PSD shows how the noise power varies over the frequency. The data was whitened by making the PSD flat, in that way, all frequencies contributed equally. A BPF was applied, and everything outside of the range of 30 - 250 Hz was eliminated.

As another way of visualising the data, a Q-transform plot was plotted. An overview of the process is shown in figure 4.

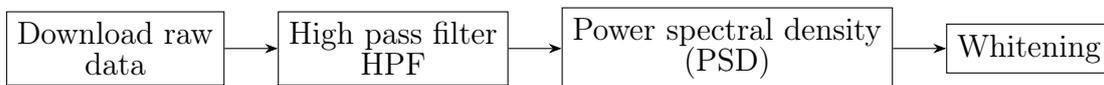


Figure 4: Overview of the main steps in the algorithm.

2.2 Calculation of Mass

The mass of the two black holes was approximated with the equations (5) and (7). The velocity in which the black holes rotate around one another, see figure 5, was approximated by Hubble's law,

$$v = H_0 D_{Mpc}, \quad (7)$$

where D stands for the distance to the merging black holes from Earth in megaparsecs (Mpc), was defined as D_{Mpc} . The unit of velocity, km/s was converted into the desired unit m/s.

In this simplified model it is assumed that the masses of the two black holes m_1 and m_2 are equal, $m_1 = m_2 = m_{1,2}$. An approximation the mass $m_{1,2}$ was calculated with

$$m_{1,2} \approx \frac{hD_{meter}c^4}{2Gv^2} \quad (8)$$

which was derived from equation (5), see appendix E, equation (13).

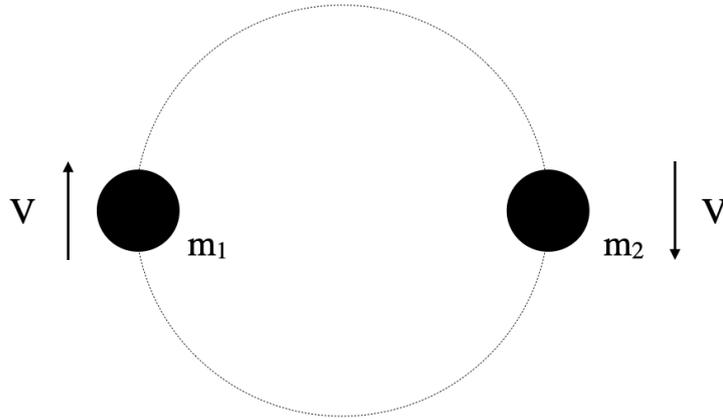


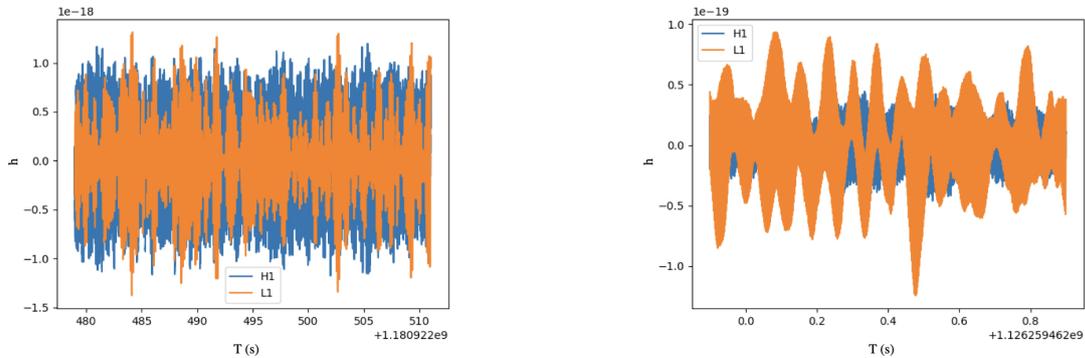
Figure 5: A simplified model of the merging of two black holes.

3 Results

The results are divided into two parts. The first part shows intermediate results in the data processing pipeline, and the second part presents the calculated masses.

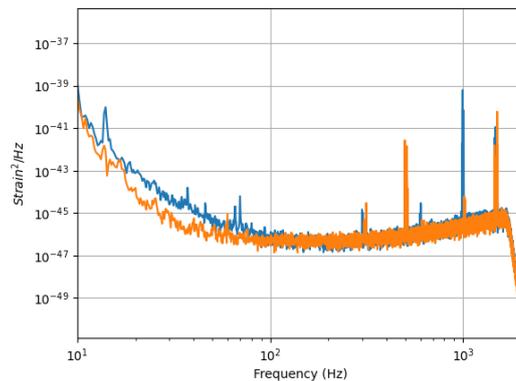
3.1 Graphs from Data Analysis

Figure 6 consists of three graphs. Figure 6a visualises the raw data from LIGO. Figure 6b visualises the raw data with a HPF. Figure 6c visualises the power spectral density of the data.



(a) The x-axis represents the time and the y-axis represents the strain. The raw data is plotted, i.e. the signal caused by two gravitational waves as well as the noise.

(b) The x-axis represents the time and the y-axis represents the strain. A HPF was applied to the data to remove the low frequency content.



(c) The x-axis represents the frequencies and the y-axis represents the $\text{strain}^2 \text{Hz}^{-1}$. It is a PSD plot of the raw data.

Figure 6: A compilation of the graphs generated by the filtering methods.

Figure 7 shows a rescaled plot of the signals with a BPF, PSD, and whitening filter. Figure 8a and 8b shows a Q-transform plot of the two detectors, H1, and L1.

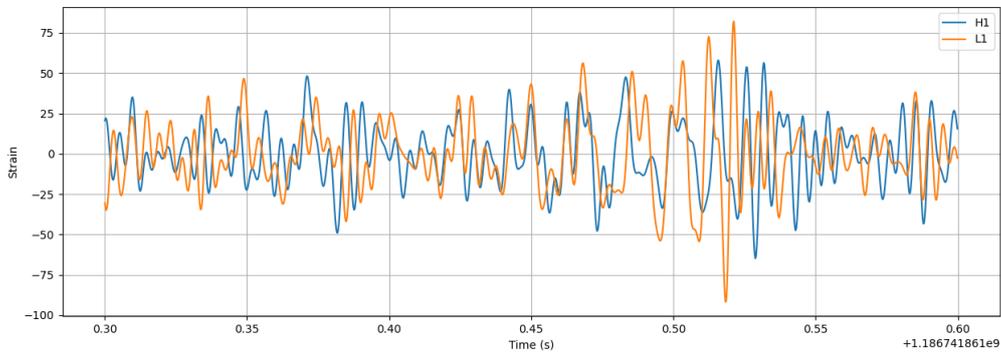
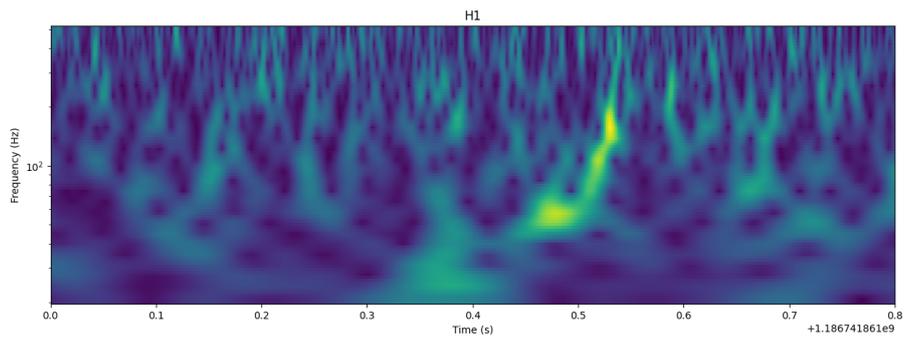
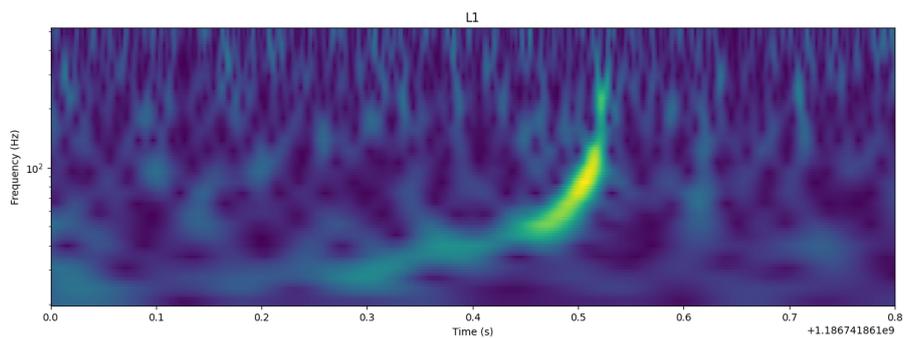


Figure 7: The x-axis represents the time and the y-axis represents the whitened strain. The plot shows the final whitened, filtered and rescaled version of the data.



(a) A Q-transform plot of the data from H1.



(b) A Q-transform plot of the data from L1.

Figure 8: A Q-transform plot of the data from L1 and H1 where the x-axis represents the time and the y-axis represents the frequency. The brighter the colour the higher the normalised energy.

3.2 Determining the Mass

From equation (5) the value of v was calculated to be

$$v = 4.95 \cdot 10^7 \text{ms}^{-1} \quad (9)$$

and the calculations can be found in appendix E, equation 12.

The values extracted from the processed data, which can be found in table 1 in appendix D, were used to make an approximation of the mass of the black holes,

$$m_{1,2} \approx 2.90 \cdot 10^5 M_{\odot}. \quad (10)$$

The calculations can be found in appendix E, equation 15.

Standard values of constants and Earth-BH distance, which can be found in table 2 in appendix D, were used to calculate the mass of the black holes,

$$m_{1,2} \approx 104 M_{\odot}. \quad (11)$$

The calculations can be found in appendix E, equation 14.

4 Discussion

Figure 7 shows a peak in the frequency when $T \approx 0.52$ in L1 and H1 also shows a peak, however, much smaller and shifted to the right relative to L1. This could be caused by imprecise measurements due to the weak strains. This uncertainty results in a significant margin of error. Additionally, there was a significant margin of error in the distance to the merging black holes and Hubble's constant. Furthermore, another source of error could be the fact that the value of the strain was determined by measuring by hand and not a computer, resulting in inexact measurements. If the value of the strain would be determined from the graphs, it would result in a mass of $2.90 \cdot 10^5 M_{\odot}$. Therefore,

calculations based on earlier approximations were used, which can be found in table 2 in appendix D, in order to receive a more accurate approximation. The final approximation was approximately four times larger than the earlier approximations, which would be acceptable considering it being astronomical distances. Moreover, equation (5) is imprecise and simplified. There exist more complex equations that can be used to calculate the masses with a smaller error margin.

4.1 Further Research and Application

In future studies, it would be interesting to compare the results when using other parameters for noise cancelling filter. Finding the filter or combination of filters that give the most exact values relative to earlier measurements.

Restrictions regarding imprecise measurements can be avoided with improved instruments studying gravitational waves. A space-based gravitational wave detector, called Laser Interferometer Space Antenna (LISA), is being developed by NASA. The arms would be 625 000 times longer than LIGO's which would result in a broader operating range. Additionally, locating the instrument in space reduces the influence of noise from terrestrial sources.[23]

It would also be of interest to find wider uses of GW measurements. This paper shows the possibility of approximating the mass of two colliding black holes, but, the signals may be able to determine other parameters of black holes in the future.

4.2 Conclusion

The filtering methods of the raw data used in this paper could be used to show the existence of the merging of two black holes. However, the method employed resulted in significant uncertainty in the estimated mass.

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Appendices

A Visualising Raw Data

The Python code from PyCBC was used to visualise the raw data from GW170814.

```
from pycbc.catalog import Merger
import pylab

m = Merger('GW170814')

data = {}
for ifo in ['H1', 'L1']:
    data[ifo] = m.strain(ifo)

for ifo in data:
    high_data = data[ifo].highpass_fir(15, 512)
    zoom = high_data.time_slice(m.time - 0.5, m.time + 0.5)
    pylab.plot(zoom.sample_times, zoom, label=ifo)

pylab.legend()
pylab.show()
```

B Q-transform

The Python code from PyCBC was used to make a Q-transform plot from GW170814.

```
import sys

from pycbc.catalog import Merger
```

```

import pylab

m = Merger('GW170814')

data = {}

for ifo in ['H1', 'L1']:
    data[ifo] = m.strain(ifo)

whitened = {}

for ifo in data:
    whitened[ifo] = data[ifo].whiten(4, 4)

    zoom = whitened[ifo].time_slice(m.time - 0.5, m.time + 0.5)
    pylab.plot(zoom.sample_times, zoom, label=ifo)

for ifo in whitened:
    zoom = whitened[ifo].time_slice(m.time - 5, m.time + 5)
    times, freqs, power = zoom.qtransform(.001, logfsteps=100,
                                         qrange=(8, 8),
                                         frange=(20, 512),
                                         )

    pylab.figure(figsize=[15, 5])
    pylab.ylabel('Frequency (Hz)')
    pylab.xlabel('Time (s)')
    pylab.pcolormesh(times, freqs, power ** 0.5)
    pylab.xlim(m.time - 0.5, m.time + 0.3)
    pylab.title(ifo)
    pylab.yscale('log')

```

```
pylab.show()
```

C Whitening Data

The Python code from PyCBC was used to whiten the raw data from GW170814.

```
from pycbc.catalog import Merger
import pylab

m = Merger('GW170814')

data = {}
for ifo in ['H1', 'L1']:
    data[ifo] = m.strain(ifo)

whitened = {}

for ifo in data:
    whitened[ifo] = data[ifo].whiten(4, 4)

    zoom = whitened[ifo].time_slice(m.time - 0.5, m.time + 0.5)
    pylab.plot(zoom.sample_times, zoom, label=ifo)

pylab.figure(figsize=[15, 3])
for ifo in whitened:
    bpsd = whitened[ifo].highpass_fir(30, 512).lowpass_fir(250, 512)

    zoom = bpsd.time_slice(m.time - 0.2, m.time + .1)
    pylab.plot(zoom.sample_times, zoom, label=ifo)
```

```

pylab.grid()
pylab.legend()
pylab.show()

```

D Input Values for Calculations

Table 1 shows the standard values approximated by LIGO, that was used to calculate the mass of the black holes. [24]

Table 1: The values that were used for the calculations using the earlier approximated value of h .

Distance (D_{Mpc})	670 Mpc
Distance (D_{meter})	$1.67 \cdot 10^{25}$ m
Gravitational constant (G)	$6.674 \cdot 10^{-11} \text{Nm}^2 \text{kg}^{-2}$
Hubble's constant (H_0)	$73.9 \text{ km s}^{-1} \text{Mpc}^{-1}$
Strain (h)	$1.40 \cdot 10^{-18}$

Table 2 shows the approximated value of h obtained from the data analysis and standard values approximated by LIGO, that was used as another way of calculating the mass of the black holes. [24]

Table 2: The values that were used for the calculations with an approximate value of h obtained from the data analysis.

Distance (D_{Mpc})	670 Mpc
Distance (D_{meter})	$1.67 \cdot 10^{25}$ m
Gravitational constant (G)	$6.674 \cdot 10^{-11} \text{Nm}^2 \text{kg}^{-2}$
Hubble's constant (H_0)	$73.9 \text{ km s}^{-1} \text{Mpc}^{-1}$
Strain (h)	$5.00 \cdot 10^{-22}$

E Calculations of Mass

With Hubble's law was the velocity calculated, see the following equations,

$$\begin{aligned}
 v &= H_0 \cdot D \\
 v &= 73.9 \cdot 670 \text{ km/s} \\
 v &= 49513 \text{ km/s} \\
 v &= 4.9513 \cdot 10^7 \text{ m/s.}
 \end{aligned} \tag{12}$$

Simplifying the equation (5) to factor out m ,

$$\begin{aligned}
 \frac{2Gm_{1,2}}{c^2} \cdot \frac{1}{D_{meter}} \cdot \left(\frac{v}{c}\right)^2 &\approx h \\
 \frac{2Gm_{1,2}}{c^2} \left(\frac{v}{c}\right)^2 &\approx D_{meter} \cdot h \\
 2Gm_{1,2} \left(\frac{v}{c}\right)^2 &\approx h \cdot D_{meter} \cdot c^2 \\
 2Gm_{1,2} v^2 &\approx h \cdot D_{meter} \cdot c^4 \\
 m_{1,2} &\approx \frac{h \cdot D_{meter} \cdot c^4}{2Gv^2}.
 \end{aligned} \tag{13}$$

From table 2 was the masses calculated,

$$\begin{aligned}
 m_{1,2} &\approx \frac{h \cdot D_{meter} \cdot c^4}{2Gv^2} \\
 m_{1,2} &\approx \frac{(5 \cdot 10^{-22}) \cdot 1.67 \cdot 10^{25} \cdot c^4}{2G(4.95 \cdot 10^7)^2} \\
 m_{1,2} &\approx 5.78 \cdot 10^{35} \\
 m_{1,2} &\approx 2.90 \cdot 10^5 M_{\odot}.
 \end{aligned} \tag{14}$$

From table 1 was the masses calculated,

$$\begin{aligned} m_{1,2} &\approx \frac{h \cdot D_{meter} \cdot c^4}{2Gv^2} \\ m_{1,2} &\approx \frac{(1.4 \cdot 10^{-18}) \cdot 1.67 \cdot 10^{25} \cdot c^4}{2G(4.95 \cdot 10^7)^2} \\ m_{1,2} &\approx 2.06 \cdot 10^{32} \\ m_{1,2} &\approx 104M_{\odot}. \end{aligned} \tag{15}$$