Research Academy for Young Scientists

Determining the Energy Density of Stochastic Gravitational Waves While Considering Cosmic Friction

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Abstract

Gravitational waves are ripples through space time that are created from massive objects in movement, such as the merging of two black holes. If old enough, they can hold information about the primordial universe that we cannot access otherwise. This paper provides information about how the energy density of gravitational waves changes over time depending on cosmic friction. This was done by solving the equation for gravitational waves numerically and using three different values for friction; -0.5, 0 and 0.5. It was found that the friction term α_M changes the shape of the power spectrum as well as the total energy over conformal time. If negative friction is applied, the energy density will increase over time. If there is positive friction it will decrease, and if no friction is considered then the energy density will stay constant over time.

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Contents

1 Introduction

Today's interest of cosmology enables a golden cosmological age where new major discoveries are being made. Cosmic microwave background (CMB) has been measured with precision, the ratio between dark and ordinary matter is being measured and new as well as improved ways to understand the structure of the universe has developed [1]. In 2015, the two detectors at the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected a brief gravitational-wave signal created from the merging of two black holes [2]. Gravitational waves are ripples through space-time, caused by motion of massive objects [3]. This was the first detection of gravitational waves since Albert Einstein's prediction in 1916.

1.1 ΛCDM- Model of the Universe

The ΛCDM model uses the cosmological principle [4]. This principle states that, on a large scale, the universe is required to be both isotropic and homogeneous. Isotropy implies that everywhere you look, everything looks the same. There is no preferred direction or orientation on a large scale. This principle implies that the distribution of matter, energy, and physical properties is statistically the same regardless of the direction in which we observe. Homogenity implies that the properties of matter and energy, such as density, are the same at any given point in space at large regions. In a homogeneous universe, no specific region or location has substantially different characteristics from any other region [5]. This also implies you cannot be in the centre of the universe, it is always expanding without any center or starting position. ΛCDM explains the evolution of the universe such as expansion and agrees with high precision observations. It also relies on physics such as dark matter, dark energy and inflation of cosmos itself [4]. This model is based of the Friedmann's equations as they relate the energy content of the universe to its geometry.

Since the universe is expanding, objects which would usually be considered at rest are comoving relative to eachother and the distance between them increasing over time is given by the Hubble parameter $H(t)$ via

$$H(t) = \frac{\dot{D}(t)}{D(t)}, \tag{1}$$

where $D(t)$ is the physical distance between two objects at time t. A special case of this equation is

$$v = DH_0, \tag{2}$$

which is the Hubble law where H_0 is the Hubble constant and v is the velocity. The comoving distance $d(t)$ is defined as a measure of the distance between two objects in cosmology, taking into account the expansion of the universe. Imagine placing out two points on a piece of an elastic band. When stretched, the distance increases in absolute terms, but the relative distance to the rubber band stays the same. In this case, $d(t)$ is the relative distance. It is introduced as the physical distance divided by the scale factor $a(t)$, via

$$d(t) = \frac{D(t)}{a(t)}, \tag{3}$$

where $a(t)$ represents the expansion of the universe. The scale factor provides a quantitative measure of the expansion or contraction of the universe over time. This implies that the universe has doubled in size from a given start time when $a(t) = 2$.

1.2 Friedmann's Equations

One of Friedmann's equations is defined as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\rho \tag{4}$$

which explains the acceleration or deacceleration of the expansion of the universe. If the term $\frac{\ddot{a}}{a}$ is negative, the expansion of the universe in decreasing, whilst a positive term means that the expansion rate is increasing. The integral over time of eq (4) obtains the rate of expansion. By substituting $\dot{a}/a$ as H and neglecting the integral constant, this results in another Friedmann's equation defined as

$$H^2 = \frac{8\pi G_N}{3}\rho, \tag{5}$$

which puts the hubble parameter H as a function of the energy density ρ of the universe. It explains how the rate of expansion, the Hubble parameter, is influenced by its energy density. Gravity, represented by the gravitational constant G_N, plays a role in this relationship. The equation provides insights into the evolution of the universe based on its energy content and energy density, forming the foundation for studying cosmological dynamics and structure.

1.3 Energy Density

In our universe, small scales are dominated the weak and strong nuclear force, as well as the electromagnetic force, and the gravitational force can be neglected. However, at large scales, the gravitational force actually dominate over quantum mechanics, but somewhere in between there is a scale where they are equal. This is called the planck mass [6]. Traditionally, the planck mass is constant, but you can also let it vary. The variation in the planck mass over time is called modified cosmic friction α_M.

While the expansion of the universe continues, the energy density of the universal components alter. In early times, the universe was mainly radiation but today, the radiation fraction is almost ignored. To put things down quantitative, the energy densities are distributed as

$$\Omega_{\Lambda,0} \approx 0.684, \quad \Omega_{\mathrm{mat},0} \approx 0.316, \quad \Omega_{\mathrm{rad},0} \approx 9.267 \times 10^{-5} \tag{6}$$

where Ω_0 is the present time energy density of the three main components in our universe; radiation (rad), matter (mat) and dark energy (Λ). However, in the past, radiation and matter were both dominant elements in the universe, but these components evolve differently over time. Since there are three spacial dimensions, ordinary matter energy density should evolve as an inverse cube, in other words $\Omega_{\mathrm{mat},0} \propto a^{-3}$ [7]. If the universe was to expand by a factor of two in each direction ($a \to 2a$), the energy density of ordinary matter would drop to 2^3. In terms of radiation, energy density drops differently. If radiation is considered to be photons when the universe expands by a factor of two in each direction ($a \to 2a$), it drops by a factor of 2^3, just as matter does. But the photons are also waves, so their wave length also gets stretched to twice as long. This gets the frequency down to half of what it was before, and this phenomenon is called the gravitational redshift [8]. When the frequency drops by a factor of two, the energy does the same, according to Planck's equation $E = hf$, which says that the energy of a photon is equal to a constant, the planck constant, multiplied by the frequency of the photon. Therefore, there is an additional drop by a factor of 2 due to gravitational redshift. When adding up the exponents, different elements evolve as

$$\Omega(a) = \left(\frac{a}{a_0}^{-4}\right) \Omega_{\text{rad},0} + \left(\frac{a}{a_0}^{-3}\right) \Omega_{\text{mat},0} + \Omega_{\Lambda,0} \tag{7}$$

states. Since dark energy is associated with the energy of vacuum, the energy density $\Omega_{\Lambda,0}$ does not vary and therefore it stays constant over time [8].

A useful notion of time is the conformal time η. Imagine the universe as a rubber sheet that is being stretched uniformly in all directions. Conformal time can be likened to measuring the time passed on this rubber sheet, where distances between objects are changing due to the stretching. This is connected with the comoving distance and therefore also the scale factor. The stretching of the rubber sheet represents the expansion of the universe, and conformal time allows us to account for this expansion when measuring the separation of events. Conformal time is related to physical time t, via

$$\eta(t) = \int_{t*}^{t} \frac{dt'}{a(t')}, \tag{8}$$

where the integration limit goes from the initial time t_* to present time. This is defined such as when multiplied by the speed of light, you will get the age of the universe. It is also possible to relate the conformal time to the scalefactor a, which would give you

$$\Delta\eta = \frac{1}{H_0}\int_{a_*}^{a} \frac{da'}{a'^2\sqrt{\Omega(a')}}. \tag{9}$$

The inverse of eq 9 will give the scale factor curve $a(t)$. Gravitational waves can have a wide range of frequencies, spanning from very low frequencies associated with cosmic events like the merging of supermassive black holes to much higher frequencies associated with astrophysical phenomena like the collision of neutron stars [9]. The distribution of energy across different frequencies of gravitational waves vary. By studying the energy spectrum of gravitational waves detected by observatories like LIGO scientists can gain insights into the nature of the sources, and probe the fundamental physics involved. The equation for gravitational wave propagation in x direction and a non-expanding universe, is given by

$$\ddot{h}(x,t) - \frac{\delta^2}{\delta x^2}h(x,t) = 0, \tag{10}$$

where $h(x,t)$ is the equation for a gravitational wave. However, this only describes one wave length, but to identify all possible wave modes, a Fourier transformation must be made. If $h(x,t)$ is the wave equation for gravitational waves in position space, then that same equation in wave number space is represented by $\tilde{h}(x,t)$. The gravitational wave energy spectra is defined as

$$\Omega_{GW}(k,t) = \frac{1}{\rho(t)}\frac{1}{32\pi G_N}\langle\dot{\tilde{h}}^2(k,t)\rangle, \tag{11}$$

where $\rho(t)$ is the energy at time t. The brackets $\langle ... \rangle$ indicate average over time. Eq 11 determines the definition of gravitational wave energy density. If written in conformal time, this can be followed by

$$\tilde{h}''(k,n) + (k^2 - \frac{a''}{a})\tilde{h}(k,\eta) = 0, \tag{12}$$

. There are extensions of eq 12. If gravitational waves are introduced with modified friction and speed, eq 12 will be extended. There are different generalisations of this, but if α_M is the one considered, then the wave equation is

$$\tilde{h}''(k,\eta) + \alpha_M \mathcal{H}\tilde{h}'(k,\eta) + (k^2 - \alpha_M \mathcal{H}^2 - \frac{a''}{a})\tilde{h}(k,\eta) = 0, \tag{13}$$

where $\mathcal{H}$ is the conformal Hubble parameter, defined as the conformal time derivative of scale factor a divided by a. This is used when the desire is to elaborate on general relativity (GR), since that model does not take cosmic friction into account in terms of gravitational waves. Therefore, when setting friction term $\alpha_M = 0$, you get the wave equation for GR.

1.4 Hopes and Constraints

There is a large number of upcoming gravitational wave detectors in the near future, such as pulsar timing array (PTA). Pulsars are rapidly rotating neutron stars that emit regular pulses of electromagnetic radiation. These pulses are incredibly stable and predictable, making them excellent cosmic clocks [10]. PTAs measure these signals caused by gravitational waves and correlates them to their arrival time. They are sensitive to low frequency signals, which is preferable in primordial gravitational wave detections since they often obtain lower frequencies. This would be very promising, but there are several difficulties regarding this. For example, gravitational waves in the frequency range of nanohertz obtain small amplitudes which requires long observational periods. It is often difficult to provide data for a longer period of time, since it requires more of both technical and financial resources. However, it is still a promosing way to learn more about our universe ranging from today all the way back to the primordial universe, and it might even bring insight into the fundamental nature of gravity itself [10].

Gravitational waves offer a direct investigation of gravity models past general relativity and they are also relatively independent of any specific modified gravity model. Gravitational waves are a part of investigating beyond relativity, but besides pointing further than that it also provides with almost no underlying gravity theory. Since gravitational waves are independant of gravity models, the difficulties will lie in how to extract the exact underlying gravity theory, which is required if the goal is to understand this fundamentally.

1.5 Aim of Study

The aim of this study is to learn about how a friction term can affect the energy density of gravitational waves. This can be helpful in future studies, where the aim is to learn more about our early universe as well as the evolution of it, from the very beginning.

2 Method

To simplify the calculations, it was assumed that the friction term α_M stays constant over time. The initial conditions of the gravitational waves were not certain and therefore set relying on similar experiments. [9]. The wave equation is solved numerically using the Pencil Code [11]. Pencil Code is a tool in the field of simulations of gravitational waves. It solves the gravitational wave propagation equation numerically[9]. It was solved in one-dimensional Fourier space with 46,000 wave numbers, which ranged from 5×10^{-3}–1×10^{1} The friction term α_M is the time variation of the Planck mass, but it is also a measure of the strength of gravity. Therefore, it has different constraints than regular friction. The equation was in three different settings where α_M varied between three different values; -0.5, 0 and 0.5. These values were based on the Planck satellite values, which was on the order of 0.1 [12], so the usage of ± 0.5 was to get clearer results.

3 Results

The results are showing both the energy density over time and the energy spectrum over the wave number. In figure 1, it is shown that the three curves become similar when the wave number increases, and that the difference between lines with different friction values happens in the low wave number area. The black line in the middle indicates the value of the energy density at initial time, whilst the fluctuating lines refer to the present time energy density. Figure 2, the energy densities when $\alpha_M = \pm 0.5$ are precisely mirrored in the line where $\alpha_M = 0$.

3.1 Gravitational Wave Energy Spectrum

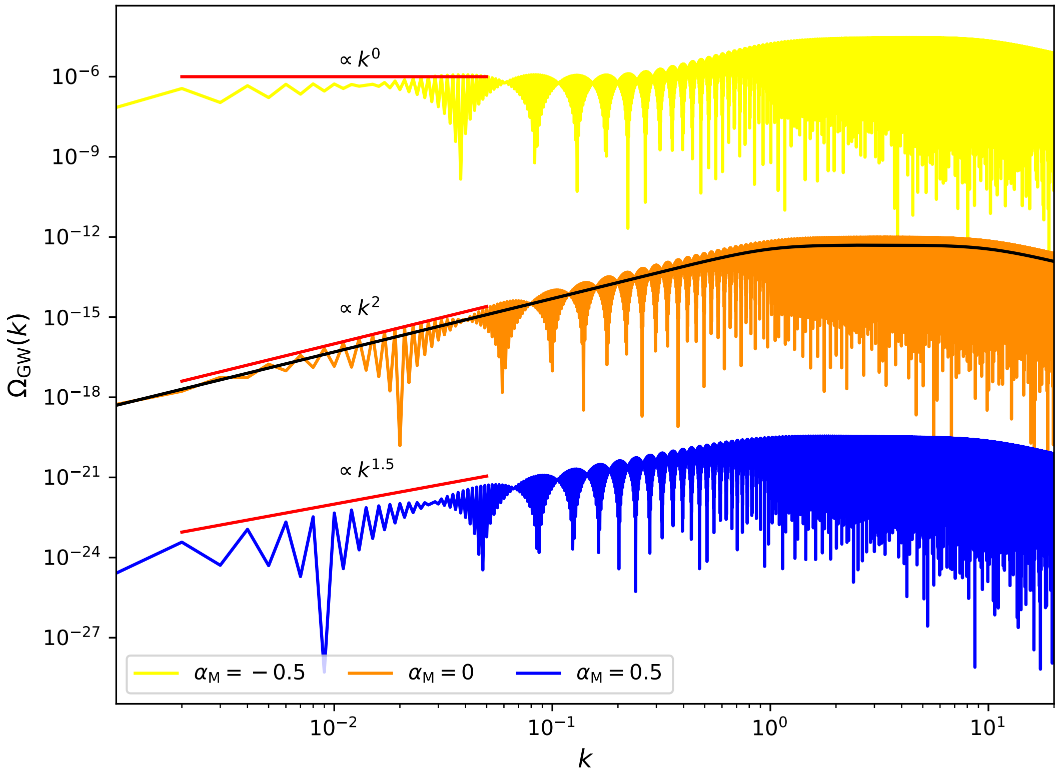

Figure 1: The middle line represents $\alpha_M = 0$, which is the case for GR, whilst the top line represent $\alpha_M = -0.5$ and the bottom line shows $\alpha_M = 0.5$.

Figure 1 shows the energy density spectrum as a function of the wavenumber k when considering three different values for the friction α_M. The black line represents the initial condition for all three variants of α_M. The other plots represent the state of energy density at present time. It is also shown, in the low wave number area, how the energy density relates to the wavenumber. This is indicated and plotted by the linear lines over each plot.

3.2 Energy Density as a Function of Conformal Time

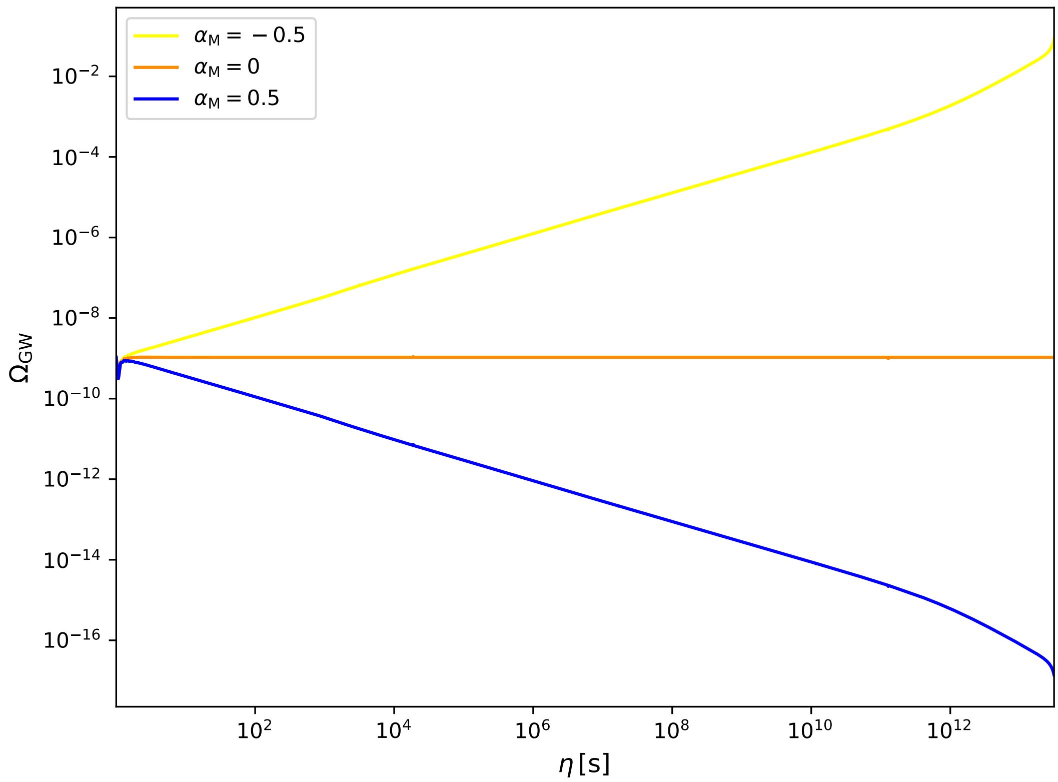

Figure 2: The total energy density of a gravitational wave as a function of conformal time

Fig 2 depends on different α_M When using $\alpha_M = 0$, the energy stays constant over time, but using $\alpha_M \neq 0$ changes the energy, either increasing or decreasing it, depending on the value of α_M. If fig 1 would be integrated over the time between the initial and a arbitrary time, it would result in that arbitrary point in fig 2.

4 Discussion

The theory of gravitational waves is an old field in science, since Albert Einstein predicted them in 1916. However, not much progress were made until the first detection of gravitational wave background in 2015 [2]. There are still constraints on today's research. The equipment used can only detect high frequency waves which means that many waves, that obtain lower frequencies, go undetected. This is a problem if the goal is to detect primordial gravitational waves, since they have travelled a longer time in space. When waves travel in space their amplitude decreases due to cosmic friction.

4.1 Result Relevance

The given results show the relevance of acknowledging the α_M as a determining factor in the wave equation. It shows how the energy density differs between the different α_M values. One implication to draw from fig 1 is that the energy density per wave number changes depending on the friction value. Since the black line represents the initial condition of all three present versions, you can look at it as an origin. When comparing the versions with the origin, you gain insight in how the energy density is affected by this. You can implicate that negative values of α_M will result in an increase of energy density per wave number, whilst a positive value will decrease the energy density. The three versions all start at different values on the y-axis. This is because they are plotted from present time, so the cosmic friction has already had time to increase or decrease the energy density. The middle spectrum is plotted with zero friction, so it follows the initial value. Another implication as to why it is not as useful to detect gravitational waves with higher wave numbers or frequencies is shown in fig 1. If the goal is to see which friction fits the best, the important difference lies in the low wave number area. If we cannot detect lower frequencies, which is the present case, then the actual changes between different friction values will go undetected.

Similar implications can be drawn from fig 2. Since it shows energy density over conformal time when considering different friction values, you can see how α_M has an impact on the energy. It is also shown that $\alpha_M = -0.5$ and $\alpha_M = 0.5$ are mirrored in $\alpha_M = 0$. The friction shifts the curve up if it is negative, and it shifts the curve down if it is positive. However, there are still some constraints to consider. Since the energy of gravitational waves are determined both by the α_M value and the initial conditions, it is difficult to decide what the actual energy should be without determining these initial conditions.

4.2 Conclusion

There has been a development in the gravitational wave field of science and there is hope that it will help us go beyond relativity, but there are still more concepts to grasp. For example, it is difficult to detect gravitational waves with lower frequencies using today's equipment, since they are only sensitive to high frequencies. This study only provided data that relied on constant α_M values. Therefore, something to do in the future would be to vary α_M and compare it. If technology improves, the hopes are that the detection of gravitational waves will provide helpful information for the understanding of fundamentals in the universe.

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