RESEARCH ACADEMY FOR YOUNG SCIENTISTS

Modelling the Effect of Cosmic Friction on Gravitational Wave Energy

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Abstract

Several different ways to modify general relativity to better match the observations have been done. One of those is to introduce what is called a cosmic friction term, α_{M}, which can be interpreted as the change of the Planck mass over time. This study investigates how the energy density and power spectrum of gravitational waves are affected by this friction term, by testing three values of α_{M}. It was found that a positive α_{M} decreases the total energy over time, while a negative α_{M} increases it. Both positive and negative friction changes the shape of the power spectrum. These results could be compared with future observations to evaluate which value of α_{M} fits best with the observations.

Acknowledgements

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Contents

1 Introduction

Albert Einstein's theory of general relativity (GR) from 1915 has made several successful predictions such as gravitational time dilation, gravitational lensing and gravitational waves. It is currently the most accurate model of gravity [1]. However, on extremely small and large scales, it is clear that it is incomplete [2]. One of the GR predictions, gravitational waves (GWs, see section 1.7), might in fact be the key to evaluating different modifications of the theory that predicted them.

1.1 Conventions

In this section, the conventions used in the paper are described. The speed of light is set to unity, i.e. $c = 1$. The derivative of a function f with respect to the physical time t is denoted $\dot{f}$, and the derivative of a function f with respect to the conformal time (see section 1.6) is denoted f'. The 0 subscript denotes the present day value, and the star subscript denotes the initial value, set to the time of the Big Bang unless otherwise stated. A tilde ($\sim$) denotes that the function is in Fourier space.

1.2 Historical Background

In 1929, Edwin Hubble proposed that the universe is expanding. He suggested a linear relationship between the distance (D) of an object and it's recessional velocity, $v = H_0 D$, with H_0 being the expansion factor, which he found was approximately $500\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$ [3]. However, he had underestimated all the distances with a factor of 7 [4], giving a result that we now know is far from reality. Since then, the expansion factor has been estimated observationally with greater accuracy. Late-universe measurements using cepheids[1] give the value of $73.4\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$ with $\sigma = 1.04\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$, but early-universe cosmic microwave background observations relying on the Lambda cold dark matter (ΛCDM) model (see section 1.5) give a value of $67.4\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$ with $\sigma = 0.5\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$, which makes a 5σ difference called the Hubble tension [2]. This is one example of the incompleteness of ΛCDM, and of GR, which is the

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[1] A cepheid is a type of star that varies in brightness in a regular manner.

theory of gravity used in ΛCDM. Modified gravity, where one or more parameters of GR is allowed to change, can play a role in extending the ΛCDM model.

1.3 Cosmic friction

The Planck mass, which is derived from the speed of light, the reduced Planck constant and the gravitational constant, is generally considered to be time independed. One way to modify gravity is to allow the gravitational constant, and therefore the Planck mass, to instead be a function of time. The rate of change of the Planck mass is called cosmic friction and denoted α_{M} [5].

1.4 Friedmann Equation

The expansion of the universe is described with the scale factor a. It scales lengths and is defined as $a_0 = 1$ corresponding to the current size of the universe, and $a(t)$ being the size of the universe at time t relative to today. The Hubble parameter $H(t)$, with the Hubble constant H_0 mentioned in section 1.2 as its current value, is defined as

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \tag{1}$$

The normalized time derivatives of the scale factor are described by the Friedmann equations, which are here simplified by assuming the large-scale curvature of the universe to be negligible (see section 1.5). The first equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho \tag{2}$$

describes the acceleration of the expansion, and the second

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G}{3}\rho \tag{3}$$

describes the expansion velocity [6]. Here, ρ is the total density of the universe.

1.5 ΛCDM

GR is a part of the ΛCDM model, which is generally considered the standard model of cosmology [7]. It will be the basis of this paper as well, but later on, modifications of this model will be introduced. The ΛCDM model assumes the cosmological principle, that is that the universe on very large scales is homogeneous and isotropic. Homogeneous means that all matter is evenly distributed throughout the universe, and isotropic means that the density has the same value in all directions. It also assumes that the universe is very flat, so that its curvature is negligibly small.

According to the ΛCDM model, the universe consists of dark energy, matter (including dark matter and ordinary matter) and radiation (mostly the cosmic microwave background, but also other electromagnetic waves and gravitational waves). The energy density is denoted with Ω, where Ω_{Λ} is the energy density of dark energy, Ω_{mat} is the energy density of matter and Ω_{rad} is the energy density of radiation. The current energy densities are estimated to be

$$\Omega_{\Lambda,0} = 0.684, \tag{4}$$

$$\Omega_{\text{mat},0} = 0.316, \tag{5}$$

$$\Omega_{\text{rad},0} = 9.267 \cdot 10^{-5}, \tag{6}$$

normalized such that the current total energy density is 1 [8].

The density of dark energy stays constant over time, since it is associated with the density of vacuum [7], which does not get diluted with an expanding universe. The density of matter decreases at the same rate as the volume increases, that is with the scale factor to the power of three. The density of radiation decreases with the scale factor to the power of four, since the photons are diluted like matter, while the wavelength is stretched, which decreases the energy of each photon as well. The function of the total energy density over time is thus

$$\Omega(a) = a^{-4}\,\Omega_{\text{rad},0} + a^{-3}\,\Omega_{\text{mat},0} + \Omega_{\Lambda,0}. \tag{7}$$

1.6 Conformal Time and Hubble Parameter

Another way to measure time, which takes the expansion of the universe into account, is the conformal time η, defined as

$$\eta(t) = \int_{t_*}^{t} \frac{\mathrm{d}t}{a(t)}. \tag{8}$$

Using the Friedmann equations, the conformal time can also be described as a function of the scale factor, which can then be numerically calculated and inverted to describe the scale factor as a function of the conformal time, $a(\eta)$ [9]. Using the conformal time, the conformal Hubble parameter $\mathcal{H}$ can now be defined as

$$\mathcal{H}(\eta) = \frac{a'(\eta)}{a(\eta)}. \tag{9}$$

1.7 Gravitational Waves

Gravitational waves are oscillations in spacetime, created by perturbations of spacetime [10]. The first direct detection of a GW was done in 2015 by the Laser Interferometer Gravitational-Wave Observatory, measuring a GW that had been produced by a binary black hole merger [11]. In June 2023, the North American Nanohertz Observatory for Gravitational Waves published their finding of the stochastic gravitational wave background, consisting of multiple binary black hole mergers and other astronomical events [12]. GWs created by cosmic fluctuations in the early universe, such as bubble nucleation from the electroweak phase transition[2] [16], have not yet been detected.

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[2] The electroweak phase transition was the separation of the weak nuclear force and the electromagnetic force, which also gave mass to electrons and quarks via the Higgs mechanism [13]. This occured at a temperature of 159.5 ± 0.5 GeV [14], corresponding to the time 0.1 ns [15]. This phase transition is thought to have happened in different bubbles throughout the universe, that then collided, nucleated, and gave rise to GWs.

1.8 Gravitational Wave Propagation

The propagation of a GW can in GR be described by the equation

$$\tilde{h}''(k,\eta) + \left(k^2 - \frac{a''(\eta)}{a(\eta)}\right)\tilde{h}(k,\eta) = 0, \tag{10}$$

where $\tilde{h}$ denotes the amplitude/strain of the gravitational wave with wavenumber k and at time η [5]. The wavenumber is defined as the inverse of the wavelength.

If a generic friction term α_M is introduced, the modified equation becomes

$$\tilde{h}''(k,\eta) + \alpha_\mathrm{M}\mathcal{H}\tilde{h}'(k,\eta) + \left(k^2 - \alpha_\mathrm{M}\mathcal{H}^2 - \frac{a''(\eta)}{a(\eta)}\right)\tilde{h}(k,\eta) = 0, \tag{11}$$

with $\mathcal{H}$ being the conformal Hubble parameter described in section 1.6 [5].

1.9 Gravitational Wave Energy Density

The GW energy density Ω_GW at wavenumber k and time η can be found by the proportionality

$$\Omega_\mathrm{GW}(k,\eta) \propto \langle \dot{\tilde{h}}^2(k,\eta)\rangle, \tag{12}$$

where the angle brackets denote the time averaged value of the curve, and the proportionality constant depends on the initial conditions of the event producing the GW. The total energy density of a GW at a time η is obtained by fixing η and integrating over k.

1.10 Aim of Study

The aim of this paper is to investigate what effect the cosmic friction has on the GW energy density over time, from their creation to the present day, to examine the current energy spectrum of gravitational waves, and to investigate what the results compared with future GW observations can tell us about the nature of gravity.

2 Method

This study was limited to modelling the effect of cosmic friction on GWs, and not in combination with other possible modifications such as the speed of gravity or the graviton mass, which was therefore set to be 1 respectively 0, in accordance with GR. It was assumed in this study that α_{M} is not time dependent. This is not because it is considered certain that this is the case, but rather to limit the complexity of the model and the calculations. Three different values of α_{M} were tested: $\alpha_{\mathrm{M}} = -0.5$, $\alpha_{\mathrm{M}} = 0$ and $\alpha_{\mathrm{M}} = 0.5$. This range was chosen because of the by previous research found constraint $|\alpha_{\mathrm{M}}| \lesssim \mathcal{O}(10^{-1})$ [5]. The value $\alpha_{\mathrm{M}} = 0$ was chosen as to have the wave function of GR as a reference, while $\alpha_{\mathrm{M}} = \pm 0.5$ were chosen to investigate if there are any symmetrical properties arising from the friction. The origins of the GW background was assumed to be the electroweak phase transition, and the initial time was thus set to be 0.1 ns. The initial conditions of the event creating the GWs (such as bubble size at the electroweak phase transition) are not completely known, and were thus set to be as in [5]. A modification of Equation (11), normalized so that the wavenumber $k = 1$ corresponds to the size of the universe at that moment[3], was solved numerically with the Pencil Code[4] , for the three different α_{M}. The obtained results were then used in Equation (12), to calculate first the total energy as a function of time, then the energy individually for the spectrum of wavenumbers from $k = 10^{-3}$ to $k = 20$ at the initial time and the current time.

3 Results

Figure 1 shows the total GW energy density as a function of conformal time, for the three different α_{M}. With no friction, the total energy density stays constant over time. With a negative friction term, the energy increases, and with a positive friction term, the energy decreases over time. These effects are symmetrical.

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[3] This means that if $k = 1$ at t, the wavelength of the GW at time t is the size of the universe at that time. Wavenumbers greater than 1 correspond to small scales, and wavenumbers smaller than 1 correspond to large scales.

[4] The Pencil Code is a code used for solving differential equations and modelling astrophysical phenomena. It can be found in [17].

Figure 2 shows the power spectrum, which is the energy density at the current point in time, as a function of a specific wavenumber. This was plotted for the three different α_M. As a reference, the power spectrum at the initial time is also plotted. The power spectra follow a double broken power law, that is, the shape of each curve can be estimated by three consecutive straight lines in a logarithmic plot.

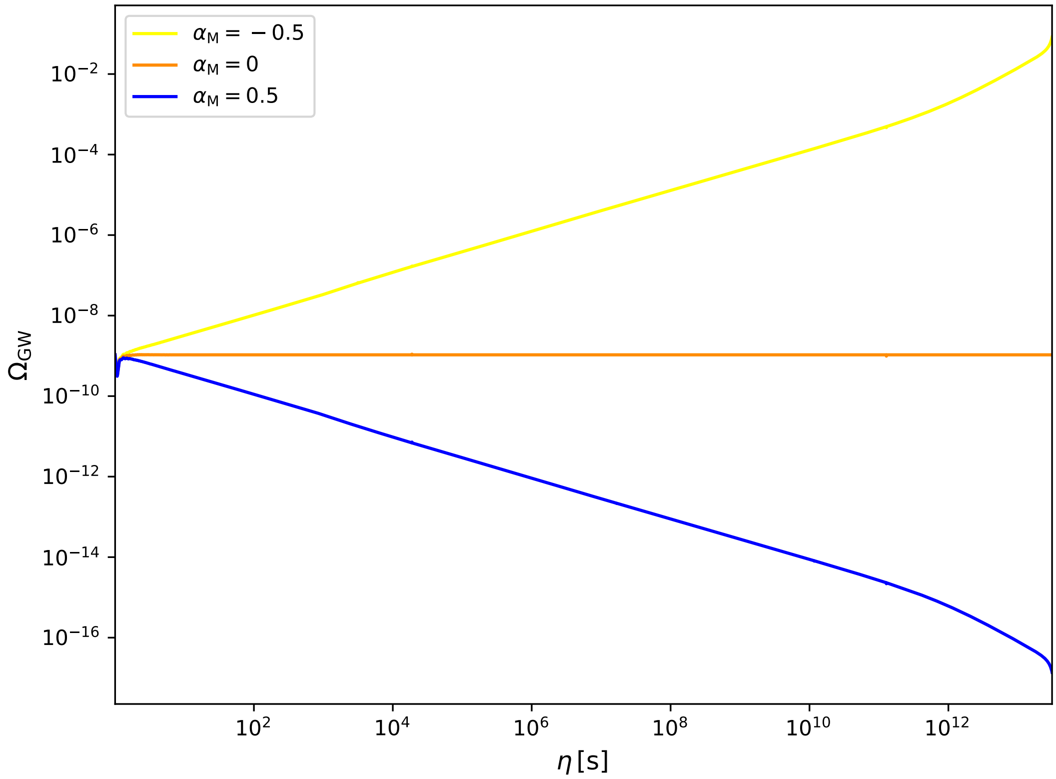

Figure 1: Time series. The total energy density of GWs as a function of time. The yellow line corresponds to $\alpha_\mathrm{M} = -0.5$, the orange to $\alpha_\mathrm{M} = 0$ and the blue to $\alpha_\mathrm{M} = 0.5$.

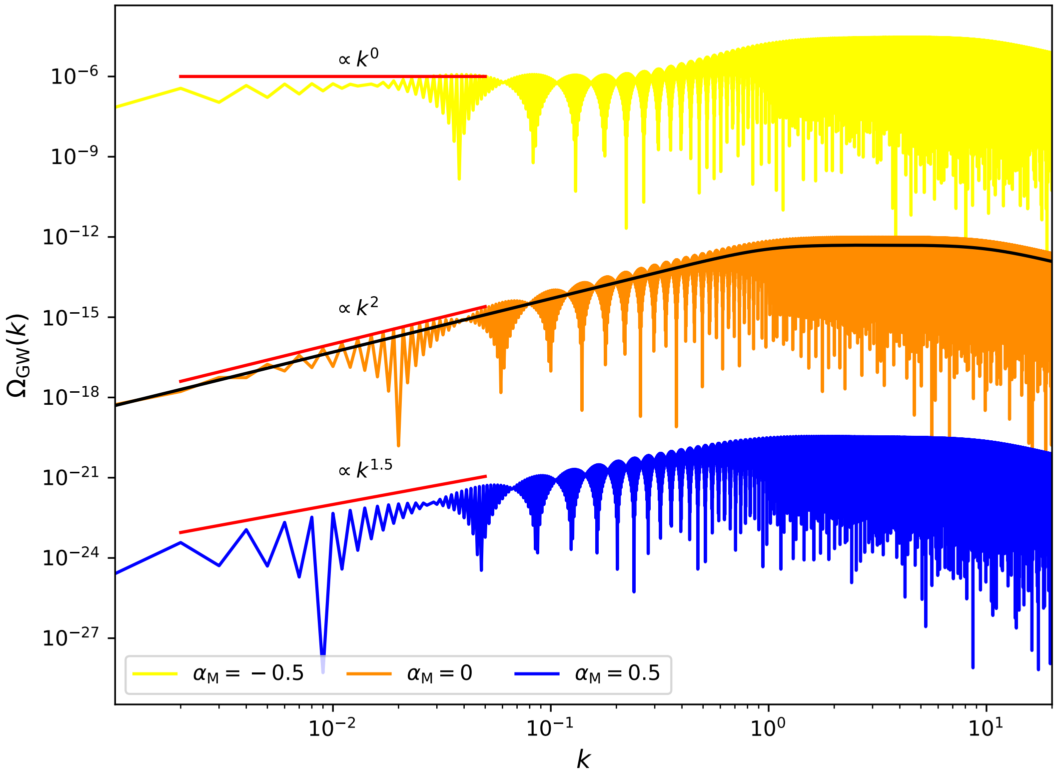

Figure 2: Power spectrum. The GW energy density of a certain wavenumber as a function of that wavenumber. The yellow line corresponds to $\alpha_{\mathrm{M}} = -0.5$, the orange to $\alpha_{\mathrm{M}} = 0$ and the blue to $\alpha_{\mathrm{M}} = 0.5$, all at the current time. The black line shows the power spectrum at the initial time, and is independent of the friction.

4 Discussion

Firstly, it is clear that the friction affects the total energy over time. A negative friction makes the energy increase, while a positive friction makes the energy decrease. This could affect how soon the gravitational waves are detected, since a higher energy leads to a greater likelihood of detection. As seen in Figure 1, a friction term of ±0.5 corresponds to a change in energy density of ±8 orders of magnitude.

When it comes to comparing the results with observations, this difference is not sufficient to determine what value α_{M} should have. This is because the current energy of the GWs is not only determined by the friction term, but also by the initial conditions of the events that produced the GWs, with at least a factor of 10^3 [18]. Since we have insufficient information about the initial conditions, we do not know which energy density that corresponds to which α_{M}. Until we have a more certain description of these early universe events, we can not use the amplitude to determine the friction.

However, another thing that changes with the friction is the shape of the spectrum. In small scales (corresponding to large wavenumbers), the shape of the spectrum is independent of the friction term, but in large scales (corresponding to small wavenumbers) the slope is determined by the friction term. In GR, the slope is proportional to k^2, but with a friction term, the exponent decreases. In contrast to the time series, the friction term does not affect the power spectrum symmetrically. This could make it more difficult to determine the friction from the shape of the power spectrum.

For all α_{M}, the spectrum follows a double broken power law. It has been suggested that the breaks occur at wavenumbers corresponding to the mean bubble separation and the thickness of the shell surrounding the bubbles [16]. However, the breaks in these calculations occur at approximately $k = 1$ and 10, which would mean that the bubble size would be the size of the universe at that time, implying that there could be no bubble collisions and therefore no source of GWs. Therefore, it is probable that the wavenumbers corresponding to the breaks mean something else. A hypothesis could be that the break at $k = 10$ corresponds to the bubble size, while the break at $k = 1$ simply corresponds to the size of the universe. To determine this, further studies are needed.

The GW background that this study examines comes from cosmic events in the early universe, and has thus been diluted with the expansion of the universe, making the energy density much lower than that of previously detected GWs from merging black holes and similar late-time events, and from the stochastic GW background detected earlier this year. Therefore, the early universe GW background has not yet been detected. However, new detectors with higher sensitivity ranges and different frequency ranges, such as the Laser Interferometer Space Antenna, the Square Kilometer Array and the Pulsar Timing Arrays, are under development and construction. Depending on the initial conditions, the energy of the GW background could possibly be high enough to be detected by one of these detectors. There are also projects trying to use the Gertsenshtein effect, which is the convertion between electromagnetic waves and gravitational waves in a magnetic field [19], to be able to use already existing optical equipment for detecting gravitational waves. When future GW detectors detect an early universe GW power spectrum, it could be possible to examine the shape of the spectrum and thereby draw conclusions of what value the friction term should have.

This study made the assumption that α_{M} is constant. However, it could as well be time dependent. Therefore, a natural following of this study is to model the time series and energy spectrum for different α_{M} functions. Another extension for future research is to vary other factors apart from friction. Modified gravity theories include not only a friction term, but also the possibility of a graviton mass and that the velocity of GWs could be different from that of light. These factors are presumed to change the power spectrum in other ways than the friction term, meaning that future analysis could evaluate all of these factors simultaneously.

References

[1] E. Berti, E. Barausse, V. Cardoso, L. Gualtieri, P. Pani, U. Sperhake, L. C. Stein, N. Wex, K. Yagi, T. Baker, *et al.*, "Testing general relativity with present and future astrophysical observations," *Classical and Quantum Gravity*, vol. 32, no. 24, p. 243001, 2015.

[2] L. Perivolaropoulos and F. Skara, "Challenges for λcdm: An update. arxiv 2021," *arXiv preprint arXiv:2105.05208*.

[3] E. Hubble, "A relation between distance and radial velocity among extra-galactic nebulae," *Proceedings of the national academy of sciences*, vol. 15, no. 3, pp. 168–173, 1929.

[4] R. P. Kirshner, "Hubble's diagram and cosmic expansion," *Proceedings of the National Academy of Sciences*, vol. 101, no. 1, pp. 8–13, 2004.

[5] Y. He, A. R. Pol, and A. Brandenburg, "Modified propagation of gravitational waves from the early radiation era," *Journal of Cosmology and Astroparticle Physics*, vol. 2023, no. 06, p. 025, 2023.

[6] A. Friedmann, "The friedmann equations," 1999.

[7] N. Aghanim, Y. Akrami, F. Arroja, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. Barreiro, N. Bartolo, *et al.*, "Planck 2018 results-i. overview and the cosmological legacy of planck," *Astronomy & Astrophysics*, vol. 641, p. A1, 2020.

[8] N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. Banday, R. Barreiro, N. Bartolo, S. Basak, *et al.*, "Planck 2018 results-vi. cosmological parameters," *Astronomy & Astrophysics*, vol. 641, p. A6, 2020.

[9] University of Oslo, Faculty of Mathematics and Natural Sciences, "Equation summary, Exam for AST5220/9420 — Cosmology II." Available online, 2016.

[10] K. S. Thorne, "Gravitational waves," *arXiv preprint gr-qc/9506086*, 1995.

[11] B. P. Abbott, R. Abbott, T. Abbott, M. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. Adhikari, *et al.*, "Observation of gravitational waves from a binary black hole merger," *Physical review letters*, vol. 116, no. 6, p. 061102, 2016.

[12] G. Agazie, A. Anumarlapudi, A. M. Archibald, Z. Arzoumanian, P. T. Baker, B. Bécsy, L. Blecha, A. Brazier, P. R. Brook, S. Burke-Spolaor, *et al.*, "The nanograv 15 yr data set: Evidence for a gravitational-wave background," *The Astrophysical Journal Letters*, vol. 951, no. 1, p. L8, 2023.

[13] G. Bernardi, M. Carena, and T. Junk, "Higgs bosons: Theory and searches," November 2007.

[14] M. D'Onofrio and K. Rummukainen, "Standard model cross-over on the lattice," *Physical Review D*, vol. 93, no. 2, p. 025003, 2016.

[15] D. Gorbunov, "Cosmology: Lecture 1 present universe: main ingredients and the expansion law." Available online, 2014.

[16] M. Hindmarsh and M. Hijazi, "Gravitational waves from first order cosmological phase transitions in the sound shell model," *Journal of Cosmology and Astroparticle Physics*, vol. 2019, no. 12, p. 062, 2019.

[17] A. Brandenburg, A. Johansen, P. A. Bourdin, W. Dobler, W. Lyra, M. Rheinhardt, S. Bingert, N. E. L. Haugen, A. Mee, F. Gent, *et al.*, "The pencil code, a modular mpi code for partial differential equations and particles: multipurpose and multiuser-maintained," *arXiv preprint arXiv:2009.08231*, 2020.

[18] A. Brandenburg, T. Kahniashvili, S. Mandal, A. R. Pol, A. G. Tevzadze, and T. Vachaspati, "Evolution of hydromagnetic turbulence from the electroweak phase transition," *Physical Review D*, vol. 96, no. 12, p. 123528, 2017.

[19] M. Gertsenshtein, "Wave resonance of light and gravitional waves," *Sov Phys JETP*, vol. 14, pp. 84–85, 1962.